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# Erratum and Addendum: New Approach to Einstein–Petrov Type I Spaces. II. Classification Scheme[1]

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There is an error in the proof of Theorem 3(c); equation (35) does not follow from equation (32) as stated. However the result in Theorem 3(c) does hold, but for a smaller subclass of Class C spaces than claimed. A more appropriate subdivision of these spaces, with corrected statement and proof of Theorem 3, is given below.

The natural subdivision of Class C spaces is:

(a) $A_\mu + kB_\mu - 2C_\mu = 0$, where $k$ is a constant.
(b) $pA_\mu + qB_\mu + rC_\mu = 0$, where $(p/r)$ and $(q/r)$ are analytic functions of $\Psi_0$ and $\Psi_2$.
(c) $pA_\mu + qB_\mu + rC_\mu = 0$, where $(p/r)$ and $(q/r)$ are analytic functions of $\Psi_0$, $\Psi_2$, and $\Phi$; $\Phi$ is a function of $\bar{\Psi}_0$ and $\bar{\Psi}_2$.
(d) Other than (a), (b), and (c).

These subclasses have the following properties:

*Theorem 3.*

(a) For Class C(a) spaces $A_\mu$, $B_\mu$, and $C_\mu$ are gradient vectors, i.e.,

$$A_\mu = A_{,\mu}, \qquad B_\mu = B_{,\mu}, \qquad C_\mu = C_{,\mu}$$

(b) For Class C(b) spaces $A_\mu$, $B_\mu$, and $C_\mu$ are hypersurface orthogonal vectors, i.e.,

$$A_\mu = aA_{,\mu}, \qquad B_\mu = bB_{,\mu}, \qquad C_\mu = cC_{,\mu}$$

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(c) For Class C(c) spaces $A_\mu$, $B_\mu$, and $C_\mu$ have the form

$$A_\mu = aA_{,\mu} + \alpha_{,\mu}$$

$$B_\mu = bB_{,\mu} + \beta_{,\mu}$$

$$C_\mu = cC_{,\mu} + \gamma_{,\mu}$$

*Proof of (c).* Substitution of[3]

$$pA_\mu + qB_\mu + rC_\mu = 0 \tag{23}$$

into (6) gives

$$3\Psi_2\{rA_{[\mu;\nu]}\} + \Psi_0\{rB_{[\mu;\nu]} + 2(2p+r)A_{[\mu}B_{\nu]}\} = 0 \tag{24}$$

$$\begin{aligned} &3\Psi_2\{rB_{[\mu;\nu]} - 2(2p+r)A_{[\mu}B_{\nu]}\} \\ &\quad + \Psi_0\{(r-4p)A_{[\mu;\nu]} - 4qB_{[\mu;\nu]} - 4r(p/r)_{,[\nu}A_{\mu]} \\ &\quad - 4r(q/r)_{,[\nu}B_{\mu]}\} = 0 \end{aligned} \tag{25}$$

Since $(p/r)$ and $(q/r)$ are functions of $\Psi_0$, $\Psi_2$, and $\Phi(\bar{\Psi}_0, \bar{\Psi}_2)$, equation (25) has the form, after using equations (5),

$$\begin{aligned} &3\Psi_2\{rB_{[\mu;\nu]} - 2(2p+r)A_{[\mu}B_{\nu]}\} \\ &\quad + \Psi_0\{(r-4p)A_{[\mu;\nu]} - 4qB_{[\mu;\nu]} + rfA_{[\mu}B_{\nu]} \\ &\quad + rgA_{[\mu}\Phi_{,\nu]} + rhB_{[\mu}\Phi_{,\nu]}\} = 0 \end{aligned} \tag{25'}$$

where $f$, $g$, and $h$ are functions of $\Psi_0$, $\Psi_2$, and $\Phi(\bar{\Psi}_0, \bar{\Psi}_2)$.

With the choice of $G_{\mu\nu}$ as

$$G_{\mu\nu} = rfA_{[\mu}B_{\nu]} + rgA_{[\mu}\Phi_{,\nu]} + rhB_{[\mu}\Phi_{,\nu]} \tag{26}$$

and noting

$$G_{[\mu\nu}A_\lambda B_{\rho]} = 0 \tag{27}$$

$$G_{[\mu\nu}G_{\rho\lambda]} = 0 \tag{28}$$

the rest of the proof, from equation (29) to the end, is now valid, for the redefined class C(c) spaces.

Finally, it is noted that there is no obvious simplification in the form of the vectors for the subclass C(d) spaces.

[3]Equation numbers refer to original paper.